X100/301

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH
QUALIFICATIONS
AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin (A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point $(-1, 3)$ and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

   (*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
   Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

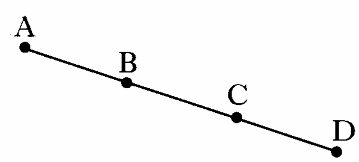
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

   (*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

   (*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

6. A and B are the points $(-1, -3, 2)$ and $(2, -1, 1)$ respectively.
   B and C are the points of trisection of AD, that is AB = BC = CD.
   Find the coordinates of D. **3**



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

8. Find $\displaystyle\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

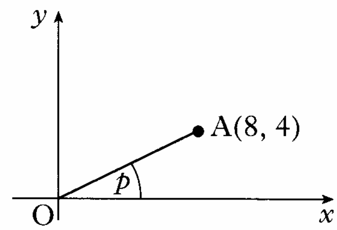
   (*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

   (*b*) Write down any restriction on the domain of h. **1**

[Turn over for Questions 10 to 12 on *Page four*

Marks

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

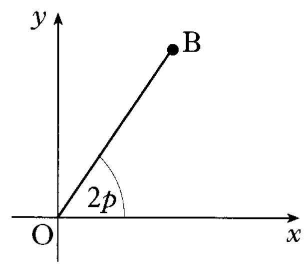


(*a*) Find the exact values of:

(i) $\sin(2p)$;

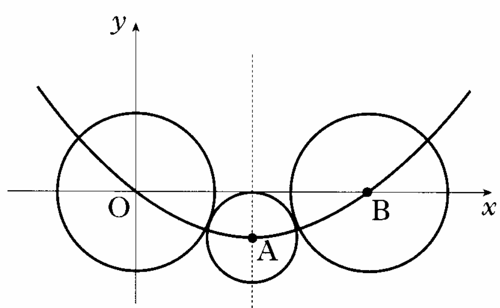
(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q. **2**

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

[*END OF QUESTION PAPER*]

X100/303

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

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ALL questions should be attempted.

Marks

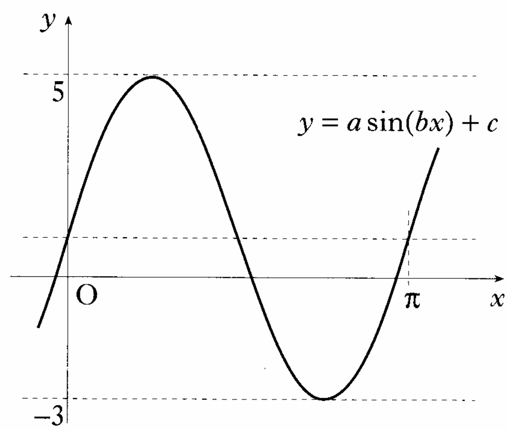
1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

   (*a*) Show that $(x - 2)$ is a factor of $f(x)$.

   (*b*) Express $f(x)$ in its fully factorised form. **4**

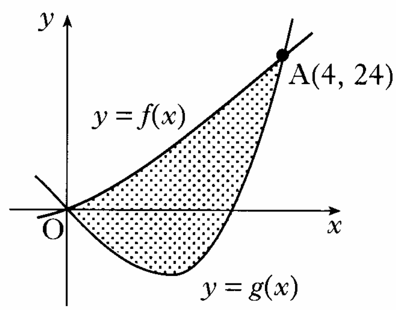
2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

   Determine the values of a, b and c. **3**



3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

   Find the shaded area enclosed between the curves. **5**



4. (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

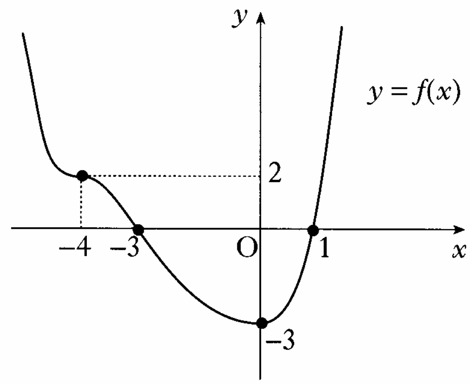
   (*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

[Turn over

Marks

5. The diagram shows the graph of a function f.

   f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

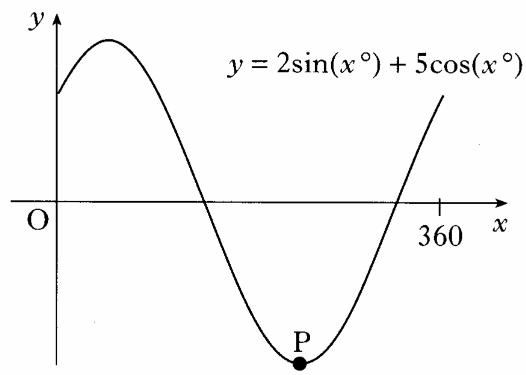


   (a) Sketch the graph of $y = f(-x)$. **2**

   (b) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**

6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

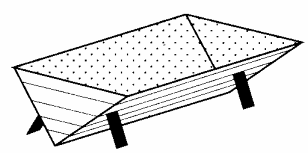
7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.



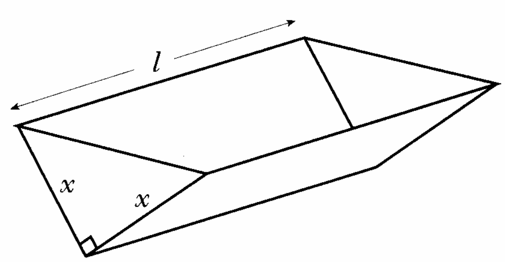
   (a) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4**

   (b) Find the coordinates of the minimum turning point P. **3**

8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



   The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.

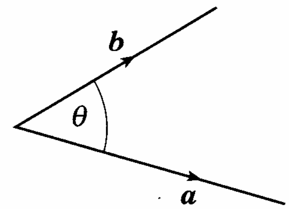


   (a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

   (b) Find the value of x which minimises this surface area. **5**

Marks

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

[*END OF QUESTION PAPER*]