Higher Solutions 2008 — Paper 1

1. $U_{11} = 0{\cdot}3 \times 10 + 6$

$\underline{U_{11} = 9}$

$U_{12} = 0{\cdot}3 \times 9 + 6$

$U_{12} = 2{\cdot}7 + 6$

$\underline{\underline{U_{12} = 8{\cdot}7}}$

$\underline{\underline{\text{Ans: C}}}$

2. Centre $= (-7, 6)$

Radius $= 6$

Equation is $(x - (-7))^2 + (y-6)^2 = 6^2$

$\underline{\underline{(x+7)^2 + (y-6)^2 = 36}}$

$\underline{\underline{\text{Ans: D}}}$

3. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$, since $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular.

$\underset{\sim}{u} \cdot \underset{\sim}{v} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 0 - 4 + k$

$= -4 + k$

let $-4 + k = 0$

$\underline{k = 4}$ $\quad \underline{\underline{\text{Ans: C}}}$

4. $L = \dfrac{b}{1-a}$

$L = \dfrac{-240}{1 - 0{\cdot}4}$

$L = \dfrac{-240}{0{\cdot}6}$

$L = -240 \times \dfrac{10}{6}$

$\underline{\underline{L = -400}}$ $\quad \underline{\underline{\text{Ans: B}}}$

5. $M_{Radius} = \dfrac{4}{5}$ $\qquad M_{Rad.} \times M_{Tang.} = -1$

$M_{Tangent} = -\dfrac{5}{4}$

Using $(7, 9)$ and $m = -\dfrac{5}{4}$

$\Rightarrow \quad y - b = m(x - a)$

$y - 9 = -\dfrac{5}{4}(x - 7)$

$\underline{\underline{\text{Ans: A}}}$

6. $2\sin x - \sqrt{3} = 0$

$2\sin x = \sqrt{3}$

$\sin x = \dfrac{\sqrt{3}}{2}$

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$x = 180 - 60^\circ$

$x = 120^\circ$

$x = \dfrac{2}{3}\pi$

$\underline{\underline{\text{Ans: B}}}$

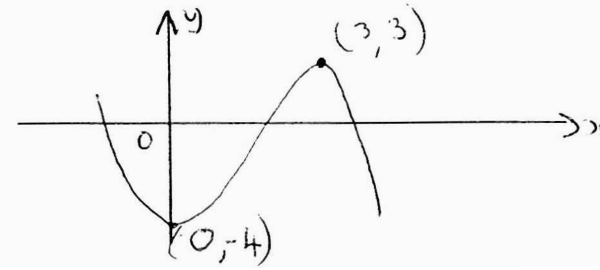
7. $m = \tan 135^\circ$

$m = -\tan 45^\circ$

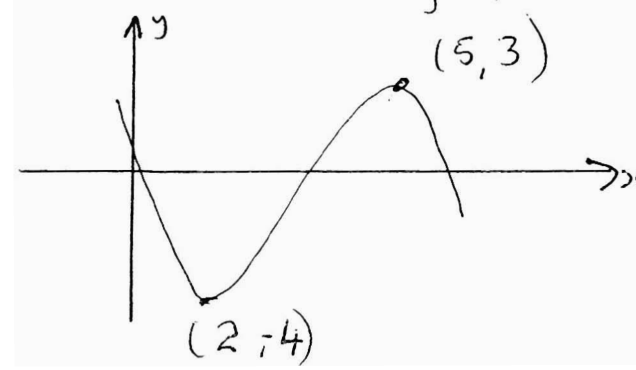
$m = -1$

Ans: C

8. $y = -f(x)$
(Reflect in x-axis)



$y = -f(x-2)$
(move to places to the right)

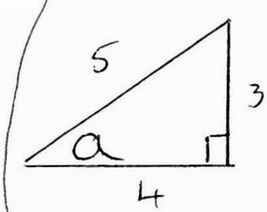


Ans: D

9. $\sin(x+a)$

$= \sin x \cos a + \cos x \sin a$

$= \sin x \times \frac{4}{5} + \cos x \times \frac{3}{5}$



$\sin a = \frac{3}{5}$

$\cos a = \frac{4}{5}$

$= \frac{4}{5}\sin x + \frac{3}{5}\cos x$

Ans: B

10. $x^2 + x + 1 = 0$

$b^2 - 4ac = 1^2 - 4 \times 1 \times 1$

$= 1 - 4$

$= -3$

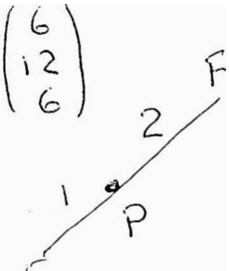
Since $b^2 - 4ac < 0$, roots are distinct but not real.

Ans: A

11. $\vec{EP} = \underset{\sim}{p} - \underset{\sim}{e} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

$\vec{PF} = \underset{\sim}{f} - \underset{\sim}{p} = \begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix}$

$\vec{EP} = \frac{1}{2}\vec{PF}$



Ans: B

12. $\overrightarrow{VT} = \overrightarrow{VW} + \overrightarrow{WS} + \overrightarrow{ST}$

$\overrightarrow{VT} = -\underline{f} + (-\underline{h}) + \underline{g}$

Ans: D

13. Roots of the equation are 1, 4

Eqn. is $y = k(x-1)(x-4)$

Using $(0,12)$ $12 = k \times (0-1)(0-4)$

$12 = 4k$

$k = 3$

Eqn. is $\underline{\underline{y = 3(x-1)(x-4)}}$

Ans: A

14. $\int 4\sin(2x+3)\,dx$

$= -\frac{1}{2} \times 4\cos(2x+3) + C$

$= -2\cos(2x+3) + C$

Ans: B

15. $y = (x^3+4)^2$

$\frac{dy}{dx} = 2(x^3+4)^1 \times 3x^2$

$\frac{dy}{dx} = 6x^2(x^3+4)$

Ans: C

16. $2x^2 + 4x + 7$

$= 2(x^2 + 2x) + 7$

$= 2[(x+1)^2 - 1] + 7$

$= 2(x+1)^2 - 2 + 7$

$= 2(x+1)^2 + 5$

Ans: A

17 To find $\sqrt{9-x^2}$,

$9 - x^2 \geq 0$

$9 \geq x^2$

$3 \geq x$ or $-3 \geq x$

$x \leq 3$ $x \geq -3$

$-3 \leq x \leq 3$

Ans: C

18. $\underline{q} \cdot (\underline{p} + \underline{q})$

$= \underline{q} \cdot \underline{p} + \underline{q} \cdot \underline{q}$

$= 10 + 4^2$

$= 26$

Ans: C

$(3, 54)$ $\quad 54 = 2 \times m^3$

$\downarrow \quad \downarrow$

$x \quad y \quad 27 = m^3$

$\underline{\underline{m = 3}}$

Ans: B

20. $y = \log_3(x-4)$

$(q, 2)$ $\quad 2 = \log_3(q-4)$

$\downarrow \quad \downarrow$

$x \quad y$

$3^2 = (q-4)$

$9 = q - 4$

$\underline{\underline{q = 13}}$ **Ans: D**

Section B

21. (a) Stat. Pts. occur when $f'(x) = 0$

$f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

let $3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1$ or $x = -1$

Using $x = 1$

$f(1) = 1^3 - 3 \times 1 + 2$

$f(1) = 0$

Stat. Pt. occurs at $(1, 0)$

Using $x = -1$

$f(-1) = (-1)^3 - 3 \times (-1) + 2$

$= -1 + 3 + 2$

$= 4$

Stat. Pt. occurs at $(-1, 4)$

Nature of Stat. Pts

	-2	-1	0	1	2
$f'(x)$	+	0	−	0	+
Shape of $f(x)$	/	—	\	—	/

Min T.P. at $(1, 0)$

Max T.P. at $(-1, 4)$

21 (b) (i)

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

Since Remainder equals zero $\Rightarrow (x-1)$ is a factor

(ii) $x^3 - 3x + 2 = (x-1)(x^2 + x - 2)$

$= \underline{\underline{(x-1)(x+2)(x-1)}}$

(c) $f(x) = x^3 - 3x + 2$

Cuts x-axis when $f(x) = 0$,

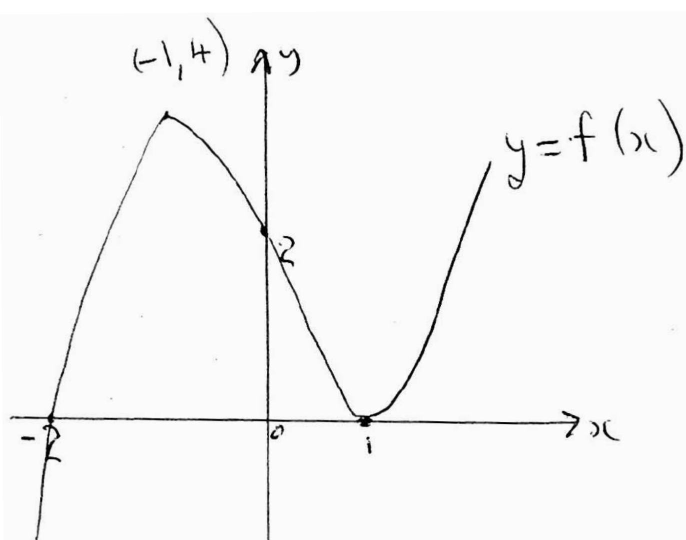
let $x^3 - 3x + 2 = 0$

$(x-1)(x+2)(x-1) = 0$ Cuts x-axis at $\underline{\underline{(1,0)\ (-2,0)}}$

Cuts y-axis when $x = 0$

$f(0) = 2$

Cuts y-axis at $\underline{\underline{(0,2)}}$



22. (a) $y = x^3 - 6x^2 + 8x$

$M_{tangent} = \frac{dy}{dx}$ $\quad \frac{dy}{dx} = 3x^2 - 12x + 8$

let $3x^2 - 12x + 8 = -1$

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$M_{tangent} = -1$ when $x = 1$ or $x = 3$

Using $x = 1$
$y = 1 - 6 + 8$
$y = 3$

Using $x = 3$
$y = 27 - 54 + 27$
$y = 0$

Points on the curve are: $(1, 3)$ $(3, 0)$

(b) $y = 4 - x$, $m = -1 \Rightarrow$ A must be $(1, 3)$ or $(3, 0)$

To find intersection between line and curve

let $x^3 - 6x^2 + 8x = 4 - x$

$x^3 - 6x^2 + 9x - 4 = 0$

$\underline{x = 1}$ $\quad 1^3 - 6\times1 + 9 - 4 = 0 \rightarrow$ Coord. of A are $\underline{\underline{(1, 3)}}$

$\underline{x = 3}$ $\quad 3^3 - 6\times3^2 + 9\times6 - 4 = 23 \Rightarrow (3, 0)$ does not lie on $y = 4 - x$

23 (a) $h(f(x)) = \log_2(x^2 - x + 10)$

$h(g(x)) = \log_2(5 - x)$

(b) $h(f(x)) - h(g(x)) = 3$

$\log_2(x^2 - x + 10) - \log_2(5 - x) = 3$

$\log_2 \frac{x^2 - x + 10}{5 - x} = 3$

$\frac{x^2 - x + 10}{5 - x} = 2^3$

$\frac{x^2 - x + 10}{5 - x} = 8$

$x^2 - x + 10 = 40 - 8x$

$x^2 + 7x - 30 = 0$

$(x + 10)(x - 3) = 0$

$x = -10$ or $x = 3$

Higher Solutions 2008

Paper 2

1. (a) $m_{BC} = \frac{-5+1}{5+3}$

$m_{BC} = -\frac{4}{8}$

$m_1 \times m_2 = -1$

$m_{Lar} = 2$

Midpoint of $BC = (1,-3)$

Eqn. of Perp. Bisector is:

$y - b = m(x-a)$

$m=2$

$(1,-3)$ $\underset{a}{\downarrow}$ $\underset{b}{\downarrow}$

$y+3 = 2(x-1)$

$\underline{y = 2x-5}$

(b) Midpoint of $AB = (2, 4)$

$m_{Median} = \frac{9}{-3}$ Using $(5,-5)$ and $(2,4)$

$m_{median} = -3$

Equation of median is:

$y - b = m(x-a)$ $m=-3$ $(5,-5)$

$y+5 = -3(x-5)$

$\underline{y = -3x+10}$

(c) Using Simultaneous Eqns:

$y = 2x-5$

$y = -3x+10$

$\Rightarrow$ $0 = 5x-15$

$x=3$

$y=1$

Point of Intersection is $\underline{(3,1)}$

2. (a) $\underline{P(8,0,4)}$

$\underline{Q(0,4,3)}$

(b) $\vec{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$

$\vec{PQ} = \begin{pmatrix}-8\\4\\-1\end{pmatrix}$

$\vec{PA} = \underset{\sim}{a} - \underset{\sim}{p}$

$\vec{PA} = \begin{pmatrix}0\\0\\-4\end{pmatrix}$

(c) $\cos Q\hat{P}A = \frac{\vec{PA}.\vec{PQ}}{|\vec{PA}||\vec{PQ}|}$

$= \frac{0+0+4}{9 \times 4}$

$= \frac{4}{36} = \frac{1}{9}$

$Q\hat{P}A = \cos^{-1}(1 \div 9)$

$\underline{Q\hat{P}A = 83 \cdot 6^\circ}$

Working $\vec{PA}.\vec{PQ} = \begin{pmatrix}0\\0\\-4\end{pmatrix}\begin{pmatrix}-8\\4\\-1\end{pmatrix}$

$= 0+0+4$

$= \underline{4}$

$|\vec{PQ}| = \sqrt{64+16+1}$ $|\vec{PA}| = \sqrt{0+0+16}$

$|\vec{PQ}| = 9$ $|\vec{PA}| = 4$

3 (a) (i) $p = \sqrt{7}$

(ii) $q = -3$

(b) $\sqrt{7}\cos x + (-3)\sin x = k\cos(x + a)$

$\sqrt{7}\cos x - 3\sin x = k\cos x\cos a - k\sin x\sin a$

$k\cos a = \sqrt{7}$

$k\sin a = 3$

$k = \sqrt{7+9} = 4$

$\tan a = \dfrac{3}{\sqrt{7}}$

$a = 48.6^\circ$

$\underline{a = 0.848 \text{ radians}}$

(c)

$f(x) + g(x) = 4\cos(x + 0.848)$

$\underline{f'(x) + g'(x) = -4\sin(x + 0.848)}$

4 (a) Centre $= (-4, -2)$

Radius $= \sqrt{16 + 4 - (-38)}$

Radius $= \sqrt{58}$

(b) Centre $= (4, 6)$

Radius $= \sqrt{26}$

Distance between Centres

$= \sqrt{(4-(-4))^2 + (6-(-2))^2}$

$= \sqrt{64 + 64}$

$= \sqrt{128} = \underline{11.3}$

Sum of 2 Radii

$= \sqrt{58} + \sqrt{26}$

$= \underline{12.7}$

Circles intersect as

$12.7 > 11.3$

(c) Using:

$x^2 + y^2 + 8x + 4y - 38 = 0$

and $y = 4 - x$

Find Pt. of Intersection:

$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$

$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x-3)(x+1) = 0$

intersects at $x = 3$ and $x = -1$

$y = 1$ $\quad y = 5$

Using $(x-4)^2 + (y-6)^2 = 26$

Put in $(3,1)$: $(3-4)^2 + (1-6)^2 = 26$

$1 + 25 = 26$

Put in $(-1,5)$: $(-1-4)^2 + (5-6)^2 = 26$

$25 + 1 = 26$

Therefore circles [illegible]

Use double angle formula

5. $\cos 2x + 2\sin x = \sin^2 x$

$1 - 2\sin^2 x + 2\sin x = \sin^2 x$

$0 = 3\sin^2 x - 2x - 1$

$0 = (3\sin x + 1)(\sin x - 1)$

$\Rightarrow \quad 3\sin x + 1 = 0 \qquad\qquad \sin x = 1$

$\sin x = -\frac{1}{3} \qquad\qquad \underline{\underline{x = 90^\circ}}$

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$x = (180 + 19.5)^\circ$

or $(360 - 19.5)^\circ$

$x = \underline{\underline{199.5^\circ \text{ or } 340.5}}$

6. (a) Find eqn. of line passing thru $(0,6)$ and $(3,0)$

$m = -2, \quad$ y-intercept $= 6$

eqn. is $\quad y = -2x + 6$

Q lies on line $\quad$ x-coord. of $Q = t$

y- coord. of $Q = -2t + 6$

$\underline{\underline{QR = 6 - 2t}}$

(b) Area of Rectangle $= t \times (6 - 2t)$

$= 6t - 2t^2$

let $\quad A(t) = 6t - 2t^2$ represent area formula

Max. Area occurs when $A'(t) = 0$

$A'(t) = 6 - 4t$

let $\quad 6 - 4t = 0$

$t = \frac{3}{2}$

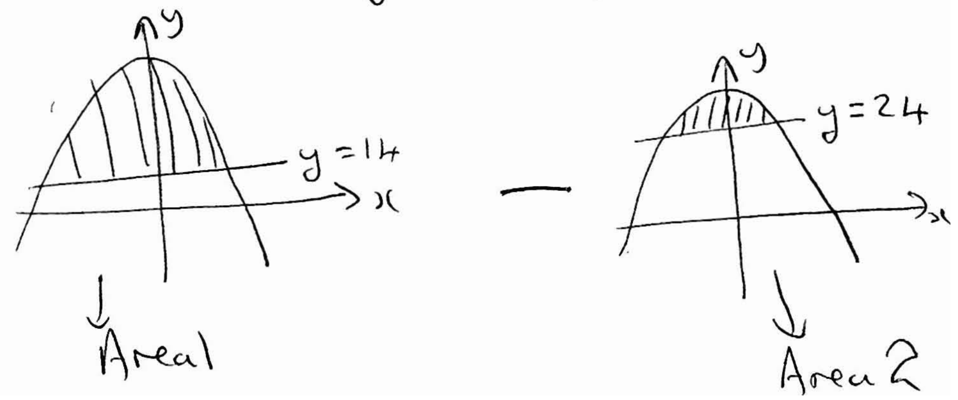
Check Nature

	1	3/2	2
A (t)	2	0	-2
A' (t)	/	—	\

Max. Area occurs when $x = 1.5$

$y = 6 - 1.5 \times 2 = 3 \qquad (1.5, 3)$

7. Shaded Area can be found by Subtracting



Calculating Area 1

Find Pts. of Intersection between curve and $y = 14$

let $32 - 2x^2 = 14$

$$18 - 2x^2 = 0$$

$$x = \pm 3$$

$$\text{Area1} = \int_{-3}^{3} 18 - 2x^2 \, dx = \left[18x - \frac{2x^3}{3} \right]_{-3}^{3}$$

$$= \left(18 \times 3 - \frac{2 \times 3^3}{3}\right) - \left(18 \times (-3) - \frac{2 \times (-3)^3}{3}\right)$$

$$= (54 - 18) - (-54 + 18)$$

$$= 108 - 36$$

$$= \underline{\underline{72}}$$

Calculating Area 2

Find Pts. of Intersection between curve and $y = 24$

let $32 - 2x^2 = 24$

$$8 - 2x^2 = 0$$

$$x = \pm 2$$

$$\text{Area } 2 = \int_{-2}^{2} 8 - 2x^2 \, dx$$

$$= \left[8x - \frac{2x^3}{3} \right]_{-2}^{2}$$

$$= \left(16 - \frac{2 \times 8}{3} \right) - \left(-16 - \frac{2 \times (-2)^3}{3} \right)$$

$$= 32 - \frac{16}{3} - \frac{16}{3}$$

$$= 21\tfrac{1}{3}$$

$$\text{Area Required} = 72 - 21\tfrac{1}{3}$$

$$= \underline{\underline{50\tfrac{2}{3}}}$$