SQA Higher Maths 2009 Paper 1

(1) $u_2 = 3u_1 + 4 = 3.2 + 4 = 10$

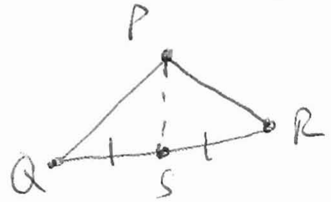
$u_3 = 3u_2 + 4 = 3.10 + 4 = 34 =$ $\boxed{A}$

(2) Radius $= \sqrt{g^2+f^2-c}$ $\quad \begin{cases} 2g = 8, & 2f = 6, \quad c = -75 \\ g = 4 & f = 3 \end{cases}$

$= \sqrt{4^2+3^2-(-75)}$ $\boxed{B}$

$= \sqrt{100} = 10$

(3)



S is mid-point of QR $= \frac{1}{2}(2, 10)$

$= (1, 5)$

$m_{PS} = \dfrac{-2-5}{-3-1} = \dfrac{-7}{-4} = \dfrac{7}{4}$ $\boxed{D}$

(4) Gradient $= \dfrac{dy}{dx} = 15x^2 - 12$ at $x = 1$ $\boxed{C}$

$= 15 - 12 = 3$

(5) (1) $ST = \sqrt{(2-5)^2 + (3-(-1))^2}$

$= \sqrt{(-3)^2 + (4)^2} = 5$ $\boxed{B}$

(2) $m_{ST} = \dfrac{3-(-1)}{2-5} = \dfrac{4}{-3}$ $\boxed{A}$

(6) Limit $= \dfrac{b}{1-a} = \dfrac{10}{1-0.7} = \dfrac{10}{0.3} = \dfrac{100}{3}$ $\boxed{A}$

(7) $\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$

$= \frac{2}{5} - 1 = -\frac{3}{5}$ $\boxed{D}$

(8) $y = \dfrac{1}{4x^3} = \dfrac{x^{-3}}{4}$, $\dfrac{dy}{dx} = \dfrac{-3x^{-4}}{4} = \dfrac{-3}{4x^4}$ $\boxed{A}$

(9) $x^2 + y^2 = 5 \Rightarrow x^2 + (2x)^2 = 5$

$5x^2 = 5$

$x^2 = 1$

$x = \pm 1$

(10) $y = \log_5(x)$ goes thro' $(5, 1)$ and $(1, 0)$ $\boxed{B}$

So $\log_5(x-2)$ is shifted 2 to the right

$= (7, 1)$ and $(3, 0)$

(11) $4\sin x - \sqrt{5} = 0$ or $\sin x + 1 = 0$

$\sin x = \sqrt{5}/4$ or $\sin x = -1$

(2 solns.) (1 soln.) $\boxed{B}$

(12) Discriminant $= b^2 - 4ac = (-1)^2 - 4(2)(-9)$

$= 1 + 72 = 73$

Real roots, distinct since $73 > 0$

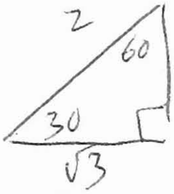
Not rational since $73 \neq$ perfect square $\boxed{C}$

(13) $\left.\begin{array}{l}\dfrac{k\sin a}{k\cos a} = \dfrac{1}{\sqrt{3}}\end{array}\right\}$ $k^2(\sin^2 a + \cos^2 a) = 1^2 + (\sqrt{3})^2$

$k^2 = 4$

So $\tan a = 1/\sqrt{3}$ $\quad k = 2$ $\boxed{B}$

$a = 30°$



(14) $\sin x$ $\left.\begin{array}{l}\text{Max} = 1\\ \text{Min} = -1\end{array}\right\}$ $2\sin(\ \ )$ Max $= 2$, Min $= -2$

So $2\sin(\ \ ) + 5$ is 7
3 $\boxed{C}$

(15) GH makes an angle $\pi/3$ with forward direction of x-axis $\boxed{A}$

Gradient $= \tan \pi/3 = \sqrt{3}$

(16) $\int_0^1 (4x^3 - 9x^2)\,dx = [x^4 - 3x^3]_0^1$ this will be negative

So the area is $-[x^4 - 3x^3]_0^1$ $\boxed{B}$

(17) $\begin{pmatrix}-3\\0\\4\end{pmatrix} = -3\underline{i} + 4\underline{k}$ Length $= \sqrt{(-3)^2 + 4^2} = 5$

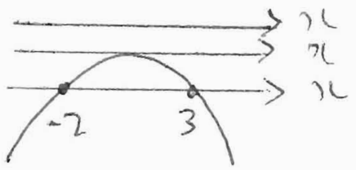
So parallel vector is $\frac{c}{5}(-3\underline{i} + 4\underline{k})$

where c is a constant $= -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{k}$ $\boxed{A}$

(18) Chain Rule $f'(x) = -\frac{1}{2}(4 - 3x^2)^{-3/2}(-6x)$ $\boxed{D}$

$= 3x(4 - 3x^2)^{-3/2}$

(19) $6 + x - x^2 < 0$



So $6 + x - x^2 < 0$ when $x < -2,\ x > 3$

$6 + x - x^2$ looks like one of the 3 possibilities

$(6 + x - x^2)$
$= (3 - x)(2 + x)$

So, $x = 3, -2$

$\boxed{C}$

(3)

(20) $A = 2\pi r^2 + 6\pi r$ Rate of change is $\frac{dA}{dr}$

So $\frac{dA}{dr} = 4\pi r + 6\pi$ when $r = 2$

$\frac{dA}{dr} = 8\pi + 6\pi = 14\pi$

$\boxed{C}$

SECTION B

(21) (a) At P, $y = 0$ So PQ is $6x - 0 + 18 = 0$

$x = -3$

$P(-3, 0)$

(b) $m_{PT} \times m_{QR} = -1$ $m_{QR} = \frac{8}{-4} = -2$

So $m_{PT} = \frac{1}{2}$

Eq. of PT is $y - (0) = \frac{1}{2}(x - (-3))$

$y = \frac{1}{2}x + \frac{3}{2}$

(c) T comes by solving $y = \frac{1}{2}x + \frac{3}{2}$ and QR equation, which is

$y - 6 = -2(x - 4)$

$y = -2x + 14$

Hence,

$-2x + 14 = \frac{1}{2}x + \frac{3}{2}$

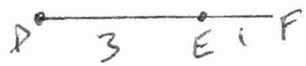
$-4x + 28 = x + 3$

$5x = 25$

$x = 5$ So $y = -2.5 + 14 = 4$ $T(5, 4)$

(22) (a) $\overrightarrow{DE} = \underset{\sim}{e} - \underset{\sim}{d} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ $\overrightarrow{DF} = \underset{\sim}{f} - \underset{\sim}{d} = \begin{pmatrix} -12 \\ 8 \\ 16 \end{pmatrix}$

$\overrightarrow{DF} = \frac{4}{3}\overrightarrow{DE}$ So D, E, F are collinear with E the common point.



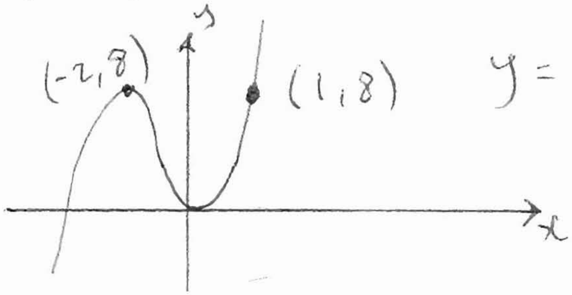
E divides DF in ratio 3 : 1

(b) $DE \perp GE$ then $\overrightarrow{DE} \cdot \overrightarrow{GE} = 0$

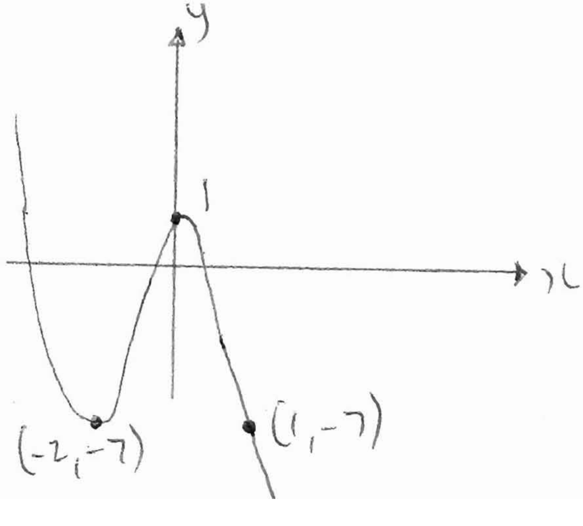
$\overrightarrow{GE} = \underset{\sim}{e} - \underset{\sim}{g} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$ Hence $\begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix} = 0$

$$-9(1-k) + 6.(-3) + 12.(-3) = 0$$
$$9k - 9 - 18 - 36 = 0$$
$$9k = 63$$
$$k = 7$$

(23) (a) $f(2x)$ = Horizontal "squeeze" by factor of 2



(b) $1 - f(2x) = -f(2x) + 1$ is $f(2x)$ graph reflected in x-axis and vertical shift by 1 up.



24 (a) $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ 60° 45°

using $\sin(A+B)$ $= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $\sin(A+B) + \sin(A-B)$

$= \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

(c) (i) $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

(ii) $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ from (a) and (c)(i)

$= \sin(A+B) + \sin(A-B)$

$= 2\sin A\cos B$ from (b)

$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \sqrt{\frac{3}{2}}$

$\left(= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}\right)$