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SQA Higher Maths Paper 2 2009

1. $y = x^3 - 3x^2 - 9x + 12$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

$\frac{dy}{dx} = 0$ for turning points

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x - 3)(x + 1) = 0$

$x - 3 = 0$ $\quad$ $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

$y = -15$ $\quad$ $y = 17$

Stationary points at $(-1, 17)$ $(3, -15)$

x	-1^-	-1	-1^+	3	3^+
$\frac{dy}{dx}$	+ve	0	−ve	0	+ve
Slope	/	—	\	—	/

when $x = -2$
$y = 15$ (+ve)

$x = 2$
$y = -10$ (−ve)

$x = 4$
$y = 15$ (+ve)

max T.Pt at $(-1, 17)$
min T.Pt at $(3, -15)$

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2. (a)(i) $f(x) = 3x+1 \qquad g(x) = x^2-2$

$$f(g(x)) = f(x^2-2)$$
$$= 3(x^2-2)+1$$
$$p(x) = 3x^2-5$$

(ii) $$g(f(x)) = g(3x+1)$$
$$= (3x+1)^2-2$$
$$q(x) = 9x^2+6x-1$$

(b) $p(x) = 3x^2-5 \qquad q(x) = 9x^2+6x-1$

$p'(x) = 6x \qquad q'(x) = 18x+6$

$$p'(x) = q'(x)$$
$$6x = 18x+6$$
$$-12x = 6$$
$$x = -\frac{1}{2}$$

3 (a)(i)

$$\begin{array}{r|rrrr} 1 & 1 & 8 & 11 & -20 \\ & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$x=1$ is a root.

(ii) $$x^3+8x^2+11x-20 = (x-1)(x^2+9x+20)$$
$$=(x-1)(x+5)(x+4)$$

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3 (b) $\log_2(x+3) + \log_2(x^2+5x-4) = 3$

$\log_2(x+3)(x^2+5x-4) = 3$

$(x+3)(x^2+5x-4) = 2^3$

$x^3+5x^2-4x+3x^2+15x-12 = 8$

$x^3+8x^2+11x-20 = 0$

$(x-1)(x+5)(x+4) = 0$

$x=1$ or $x=-5$ or $x=-4$

4. (a) $P(5,10)$ $(x+1)^2+(y-2)^2 = 100$

$LHS = (x+1)^2+(y-2)^2$

$= (5+1)^2 + (10-2)^2$

$= 6^2 + 8^2$

$= 100$

$= RHS.$

$P(5,10)$ lies on the circle

(b) $\vec{PQ} = 2\vec{PC}$ where $C(-1,2)$ is the centre of C,

$\vec{PC} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}5\\10\end{pmatrix}$

$= \begin{pmatrix}-6\\-8\end{pmatrix}$

$\vec{PQ} = \begin{pmatrix}-12\\-16\end{pmatrix}$ so $Q(5-12, 10-16)$

$Q(-7,-6)$

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4 (b) cont. $m_{PQ} = \dfrac{10-(-6)}{5-(-7)}$

$= \dfrac{16}{12}$

$= \dfrac{4}{3}$

So $m_{TAN} = -\dfrac{3}{4}$ since $m_1 m_2 = -1$

$Q(-7, -6)$

$y + 6 = -\frac{3}{4}(x+7)$

$4y + 24 = -3(x+7)$

$4y + 24 = -3x - 21$

$3x + 4y + 45 = 0$ eqn. of tangent at Q

(c) radius of circle C_1 is 10

so radii of C_2, C_3 is 20

for C_2 centre is $P(5, 10)$

for C_3 at centre, R,

$\overrightarrow{QR} = \overrightarrow{PQ}$

$= \begin{pmatrix} -12 \\ -16 \end{pmatrix}$

$\begin{pmatrix} -12 \\ -16 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -7 \\ -6 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -19 \\ -22 \end{pmatrix}$

4 (c) cont

C_2 $r=20$ Centre$(5,10)$

$$(x-5)^2+(y-10)^2=400$$

C_3 $r=20$ Centre $(-19,-22)$

$$(x+19)^2+(y+22)^2=400$$

5. (a) $g(x) = 3\cos(2x)$ $m=3,\ n=2$

(b) $f(x) = g(x)$

$$-4\cos(2x)+3 = 3\cos(2x)$$

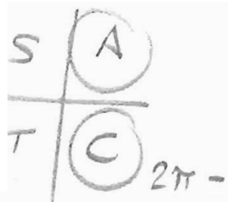
$$7\cos(2x) = 3$$

$$\cos 2x = \frac{3}{7}$$

$$2x = \cos^{-1}\left(\frac{3}{7}\right)$$

$$= 1.13,\ 5.16$$

$$x = 0.6,\ 2.6$$



Work in radians correct to 1 d.p.

at $x=0.6$

$y = 3\cos 1.13$

$= 1.3$

$x = 2.6$

$y = 3\cos 5.16$

$= 1.3$

Pts of intersection $(0.6, 1.3)$ $(2.6, 1.3)$

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5 (c) $\int_{0.6}^{2.6} -4\cos(2x) + 3 - 3\cos(2x)\ dx$

$= \int_{0.6}^{2.6} -7\cos 2x + 3\ dx$

$= \left[3x - \frac{7\sin 2x}{2}\right]_{0.6}^{2.6}$

$= \left(7.8 - \frac{7\sin 5.2}{2}\right) - \left(1.8 - \frac{7\sin 1.2}{2}\right)$

$= (7.8 + 3.1) - (1.8 - 3.3)$

$= 10.9 - (-1.5)$

$= 12.4$

6. (a) $N = N_0 e^{rt}$

$N_0 = 61$
$t = 14$
$r = 0.016$ (1.6%)

$N = 61 e^{0.016 \times 14}$
$= 76.3$ million.

(b) $N_0 = 5.1$ $r = 0.0043$
How long until $N = 2N_0$

$2N_0 = N_0 e^{0.0043t}$
$e^{0.0043t} = 2$ taking logs of both sides

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6 (b) cont $\ln e^{0.0043t} = \ln 2$

$0.0043t = \ln 2$

$t = \frac{\ln 2}{0.0043}$

$= 161.2$

Scotland's population will double in 161.2 years.

7 (a) $p.(q+r) = p.q + p.r$

$= 4 \times 3 \cos 30^\circ + 4 \times \overset{*}{\frac{3}{2}} \cos 90^\circ$

$= 12 \times \frac{\sqrt{3}}{2} + 0$

$= 6\sqrt{3}.$

* $|r| = |q| \sin 30^\circ$

$= 3 \times \frac{1}{2}$

$= \frac{3}{2}$

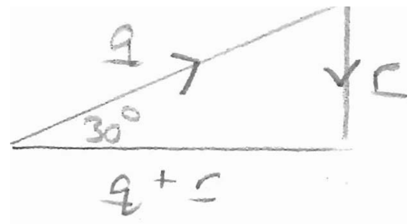
$r.(p-q) = r.p - r.q$

$= \frac{3}{2} \times 4 \cos 90^\circ - \frac{3}{2} \times 3 \cos 120^\circ$

$= 0 - \frac{9}{2}\left(-\frac{1}{2}\right)$

$\frac{9}{4}$

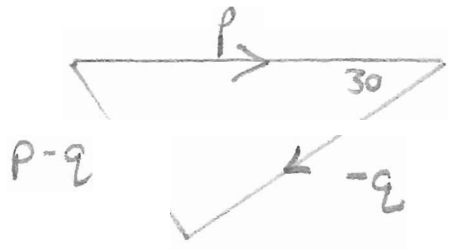
7 (b) $|q + r|$



$$\cos 30^\circ = \frac{|q+r|}{|q|}$$

$$|q+r| = 3\cos 30^\circ = \frac{3\sqrt{3}}{2}$$

$|p - q|$



using cosine rule

$$|p-q|^2 = 4^2 + 3^2 - 2 \times 4 \times 3 \times \cos 30^\circ$$

$$= 16 + 9 - 24\cos 30^\circ$$

$$= 25 - 12\sqrt{3}$$

$$|p-q| = \sqrt{25-12\sqrt{3}} \quad (2.05)$$