Higher 2010 Paper 1

1. $2x - 3y - 6 = 0$

$2x - 6 = 3y$

$\frac{2}{3}x - 2 = y$

$m = \frac{2}{3}$, $m_L = -\frac{3}{2}$

Ans: A

2. $U_1 = 2 \times U_0 + 3$

$U_1 = 2 \times 1 + 3 = 5$

$U_2 = 2 \times U_1 + 3$

$= 2 \times 5 + 3 = 13$

Ans: C

3. $3\underset{\sim}{u} - 2\underset{\sim}{v}$

$= 3\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$

Ans: D

4. $y = 2 \cos 3x$

amplitude

period $= \frac{2}{3}\pi$

Ans: A

5. $x^2 + 8x + 3$

$= (x+4)^2 - 16 + 3$

$= (x+4)^2 - 13$

Ans: B

6. Roots are equal $\Rightarrow b^2 - 4ac = 0$

$b^2 - 4ac = (-3)^2 - 4 \times k \times 2$

$= 9 - 8k$

let $9 - 8k = 0$

$k = 9/8$

Ans: D

7. $L = \frac{b}{1-a}$

$L = \frac{7}{1 - 1/4}$

$L = \frac{7}{3/4} = \cancel{7} \times \frac{4}{3}$

$= 28/3$

Ans: C

8. Centre of Circle $= (3, 5)$

Radius $= \sqrt{9 + 25 - 9} = 5$

Point on Circumference $= (3, 10)$

line is $y = 10$

Ans: B

9. $\int 2x^{-4} + \cos 5x \; dx$

$= \dfrac{2x^{-3}}{-3} + \dfrac{1}{5}\sin 5x + c$

Ans: C

10. Product of Perp. vectors is zero.

$\begin{pmatrix} x \\ 5 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} = 0$

$-3x + 10 - 7 = 0$

$-3x = -3$

$x = 1$

Ans: B

11. $f(g(\pi/6))$

$= f\left(\pi/6 + \pi/6\right)$

$= f\left(\pi/3\right)$

$= \cos \pi/3$

$= \dfrac{1}{2}$

Ans: D

12. $f(x) = x^{-1/5}$

$f'(x) = -\dfrac{1}{5} x^{-6/5}$

Ans: A

13. $a > 0 \Rightarrow$ parabola shaped $\cup$

$b^2 - 4ac > 0 \Rightarrow 2$ real distinct roots.

Ans: B

14. Area $= \displaystyle\int_{-2}^{2}$ Top Curve − Bottom Curve

$= \displaystyle\int_{-2}^{2} 14 - x^2 - (2x^2 + 2)\, dx$

$= \displaystyle\int_{-2}^{2} 12 - 3x^2 \, dx$

Ans: C

15. $f'(x) = x^2 - 9$

$x = 1$ $\quad f'(1) = 1 - 9 = -8$ (decreasing)

$x = -3$ $\quad f'(-3) = 9 - 9 = 0$ (Stationary)

Ans: C

16. $y = k(x-1)^2(x+t)$

$t = -5$ $\quad y = k(x-1)^2(x-5)$

Using $(0,10)$ $\quad 10 = k \times (0-1)^2 \times (0-5)$

$10 = k \times -5$

$k = -2$

Ans: A

17. $s(t) = t^2 - 5t + 8$

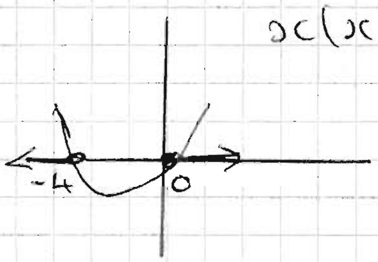
$s'(t) = 2t - 5$

$s'(3) = 6 - 5 = 1$

Ans: B

18. $x^2 + 4x > 0$

$x(x+4) > 0$



$x > 0,\ x < -4$

Ans: B

19. $y = \log x$ cuts at $(1,0)$

so graph moved $\rightarrow$ 3 units,

$y = \log(x-3)$

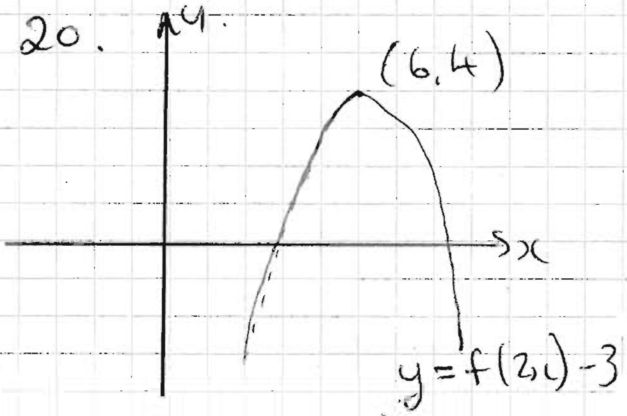
other point on $y = \log x$ must have been $(3,1)$

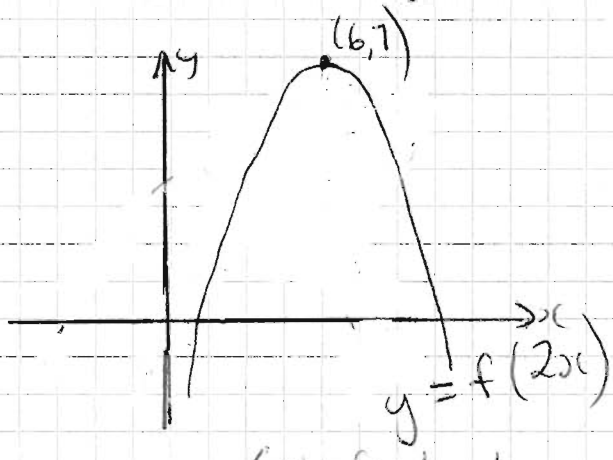
so base must be 3

$f(x) = \log_3(x-3)$

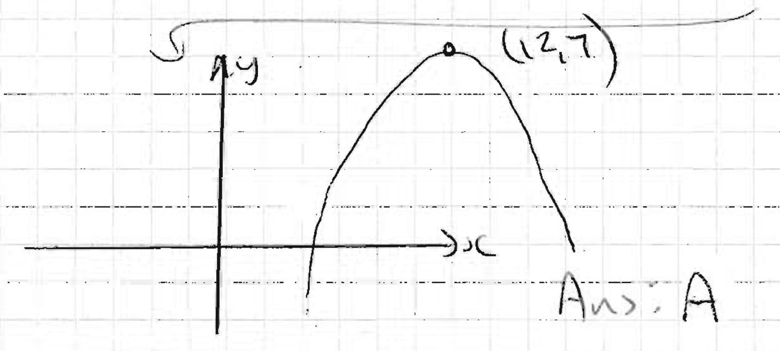
Ans: C

20.





(Shift back 3 units)



Ans: A

21. (a) $Q = \left(\frac{4+18}{2}, \frac{0+20}{2}\right)$

$Q = (11, 10)$

$M_{BQ} = \frac{-6}{15}$

Using $y - b = m(x-a)$

$y - 16 = \frac{-6}{15}(x+4)$

$15y - 240 = -6x - 24$

$6x + 15y = 216$

(b) $T\ (6, 12)$

$6x + 15y$

$= 36 + 180$

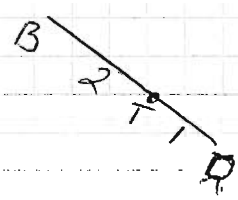
$= 216$

(c) $\vec{BT} = \underset{\sim}{t} - \underset{\sim}{b} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} - \begin{pmatrix} -4 \\ 16 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ -4 \end{pmatrix}$

$\vec{BQ} = \underset{\sim}{q} - \underset{\sim}{b} = \begin{pmatrix} 11 \\ 10 \end{pmatrix} - \begin{pmatrix} -4 \\ 16 \end{pmatrix}$

$= \begin{pmatrix} 15 \\ -6 \end{pmatrix}$



$\vec{BT} = \frac{2}{3}\vec{BQ}$

Ratio is $2:1$

22. (a) (i)

$$\begin{array}{r|rrrr} & 2 & 1 & -8 & 5 \\ 1 & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & \boxed{0} \end{array}$$

Since Remainder = 0
$(x-1)$ is a factor

(ii) $2x^3 + x^2 - 8x + 5 = (x-1)(2x^2 + 3x - 5)$

$= (x-1)(2x+5)(x-1)$

(b) $2x^3 + x^2 - 8x + 5 = 0$

$(x-1)(2x+5)(x-1) = 0$

Solution are $x = 1$, (Repeated root)
and $x = -5/2$

(c) let $2x^3 + x^2 - 6x + 2 = 2x - 3$

$2x^3 + x^2 - 8x + 5 = 0$

$(x-1)(2x+5)(x-1) = 0$

Point of Contact is at $(1, -1)$

Coord of G

$x = 1$

$y = 2 \times 1 - 3$

$y = -1$

(d) $H(-5/2, -8)$

Coords of H

$x = -5/2, \quad y = 2 \times (-5/2) - 3$

$y = -8$

23(a) (i) $\tan a = m_{OA}$

$\tan a = \dfrac{3}{2}$

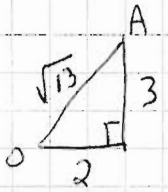
$3x - 2y = 0$

$3x = 2y$

$\frac{3}{2}x = y$

Rearrange into $y = mx + c$

(ii)



$\sin a = \dfrac{3}{\sqrt{13}}$

Using Pythagoras

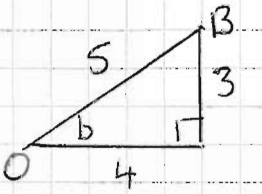
$OA = \sqrt{4+9} = \sqrt{13}$

(b) $3x - 4y = 0$

$3x = 4y$

$\dfrac{3}{4}x = y$

$m = \dfrac{3}{4}$



$\sin b = \dfrac{3}{5}$

$\cos b = \dfrac{4}{5}$

(c) (i) $\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\cos a = \dfrac{2}{\sqrt{13}}$

$= \dfrac{3}{\sqrt{13}} \times \dfrac{4}{5} - \dfrac{3}{5} \times \dfrac{2}{\sqrt{13}}$

$= \dfrac{12}{5\sqrt{13}} - \dfrac{6}{5\sqrt{13}}$

$= \dfrac{6}{5\sqrt{13}}$

(ii) $\sin(b-a) = \sin(-(a-b))$

$= -\sin(a-b)$

$= -\dfrac{6}{5\sqrt{13}}$

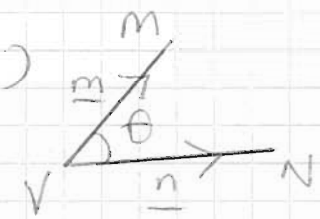
Higher 2010 Paper 2 Solutions

Q1 (a) $M(0,1,0)$ $N(4,2,2)$

(b) $\vec{VM} = \begin{pmatrix}0\\1\\0\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}0\\-1\\-3\end{pmatrix}$

$\vec{VN} = \begin{pmatrix}4\\2\\2\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}4\\0\\-1\end{pmatrix}$

(c)



$m.n = m_1n_1 + m_2n_2 + m_3n_3$

$= (0\times4) + (-1\times0) + ((-3)\times(-1))$

$= 3.$

$|m| = \sqrt{0+1+9} = \sqrt{10}$

$|n| = \sqrt{16+0+1} = \sqrt{17}$

$m.n = |m||n| \cos\theta$

$\cos\theta = \frac{m.n}{|m||n|}$

$= \frac{3}{\sqrt{10}\sqrt{17}}$

$= 0{\cdot}230$

$\theta = 76{\cdot}7^\circ$

Q2 (a) $k\cos(x+a)^\circ = k\cos a\cos x - k\sin a\sin x$

$k\cos a = 12$, $k\sin a = 5$.

$k = 13$ $\tan a = \frac{5}{12}$

$a = 22{\cdot}6^\circ$

$12\cos x^\circ - 5\sin x^\circ = 13\cos(x + 22{\cdot}6)^\circ$

(b) (i) max. value is 13

min. value is -13.

(ii) max value occurs when

$\cos(x + 22{\cdot}6)^\circ = 1$

$(x + 22{\cdot}6)^\circ = 0$ or $(x + 22{\cdot}6)^\circ = 360^\circ$

$x = -22{\cdot}6^\circ$ $\qquad$ $\underline{\underline{x = 337{\cdot}4^\circ}}$

not in interval.

min. value occurs when

$\cos(x + 22{\cdot}6)^\circ = -1$

$(x + 22{\cdot}6)^\circ = 180^\circ$

$x = 157{\cdot}4^\circ$

Q3. (a) (i) if $y = 3 - x$ is a tangent there will be only one point of contact with the circle.

$$x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$$
$$x^2 + 9 - 6x + x^2 + 14x + 12 - 4x - 19 = 0.$$
$$2x^2 + 4x + 2 = 0$$
$$2(x^2 + 2x + 1) = 0$$
$$2(x+1)^2 = 0.$$
$$(x+1) = 0$$
$$x = -1$$

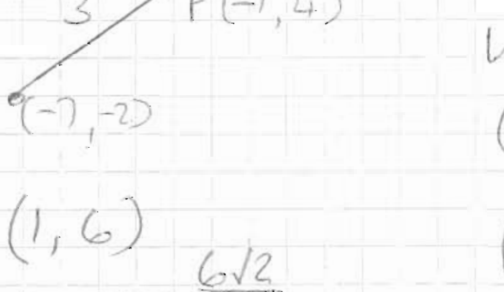
only one point of contact, so line is a tangent to the circle.

(ii) $y = 3 - x$ at $x = -1$

$y = 4$ Point of contact is $(-1, 4)$

(b) $x^2 + y^2 + 14x + 4y - 19 = 0$

Centre $(-7, -2)$ radius $= \sqrt{49+4+19}$
$= \sqrt{72}$
$= 6\sqrt{2}$



Using the section formula

$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} = \tfrac{1}{4}\begin{pmatrix} -7 \\ -2 \end{pmatrix} + \tfrac{3}{4}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 16 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \end{pmatrix} + \begin{pmatrix} 3x \\ 3y \end{pmatrix}$$

$3x - 7 = -4$ $\quad 3y - 2 = 16$

$x = 1$ $\quad y = 6$

C $(1, 6)$

radius $r = \dfrac{6\sqrt{2}}{3}$
$= 2\sqrt{2}.$

Eqn. of the circle is

$$(x-1)^2 + (y-6)^2 = 8.$$

Q4. $2\cos 2x - 5\cos x - 4 = 0$

$$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$$
$$4\cos^2 x - 2 - 5\cos x - 4 = 0$$
$$4\cos^2 x - 5\cos x - 6 = 0$$

$$(4\cos x + 3)(\cos x - 2) = 0$$

$4\cos x + 3 = 0$ $\quad$ $\cos x - 2 = 0$

$4\cos x = -3$ $\quad$ $\cos x = 2$

$\cos x = -\frac{3}{4}$ $\quad$ No solutions

$$x = 138.6°, 221.4°$$
$$= 2.42, 3.86 \text{ radians.}$$

Q5 (a) (i) $TP = x$

so $SP = 2x$ (symmetry on y-axis)

Find coordinates of T

lies on $y = \frac{2}{5}(10 - x^2)$

At T, $x = 0$

$$y = \frac{2}{5}(10 - 0^2)$$
$$= 4. \qquad T(0, 4)$$

Coordinates of P can be written in the form $(x, 4)$

Coordinates of Q can be written in the form $(x, 10 - x^2)$

So Length of $PQ = 10 - x^2 - 4$

$$= 6 - x^2$$

(ii) Area of $PQRS = PQ \times PS$

$$A = (6 - x^2) \times 2x$$
$$= 12x - 2x^3$$

Q5(b) $A(x) = 12x - 2x^3$

$A'(x) = 12 - 6x^2$

$A'(x) = 0$ for turning points

$12 - 6x^2 = 0$

$6x^2 = 12$

$x^2 = 2$

$x = \pm\sqrt{2}$

$x \neq -\sqrt{2}$ since this would lead to PS being negative

x	$\sqrt{2}^-$	$\sqrt{2}$	$\sqrt{2}^+$
$A'(x)$	+ve	0	−ve
Slope	/	—	\

maximum area occurs when $x = \sqrt{2}$.

$A = 12x - 2x^3$

$= 12\sqrt{2} - 4\sqrt{2}$

$= 8\sqrt{2}$.

Q6 (a) $y = (2x - 9)^{\frac{1}{2}}$

when $x = 9$

$y = 3$ $(9,3)$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-\frac{1}{2}} \times 2$

$= (2x-9)^{-\frac{1}{2}}$

when $x = 9$

$\frac{dy}{dx} = \frac{1}{3}$.

$y - b = m(x - a)$

$y - 3 = \frac{1}{3}(x - 9)$

$y = \frac{1}{3}x$.

6(b) At point A $y=0$

so $(2x-9)^{\frac{1}{2}} = 0$

$$2x-9=0$$
$$2x=9$$
$$x=\frac{9}{2} \qquad A\left(\frac{9}{2}, 0\right)$$

(c) Area of triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 9 \times 3$$
$$= \frac{27}{2}$$

or

$$\int_0^9 \frac{1}{3}x \, dx$$
$$= \left[\frac{1}{6}x^2\right]_0^9$$
$$= \frac{81}{6} - 0$$
$$= \frac{27}{2}$$

Find area under curve $y=(2x-9)^{\frac{1}{2}}$

$$\int_{\frac{9}{2}}^{9} (2x-9)^{\frac{1}{2}} \, dx = \left[\frac{(2x-9)^{\frac{3}{2}}}{3}\right]_{\frac{9}{2}}^{9}$$
$$= [9] - [0]$$
$$= 9$$

Shaded area, $A = \frac{27}{2} - 9$

$$= \frac{9}{2} \text{ units}^2$$

Q7 (a) $\log_4 x = p$

$4^p = x$

$\log_{16} 4^p = \log_{16} x$

$p \log_{16} 4 = \log_{16} x$ since $y = \log_{16} 4$

$\frac{1}{2} p = \log_{16} x$ $16^y = 4$

$y = \frac{1}{2}$

(b) $\log_3 x + \log_9 x = 12$

from part(a) $\log_9 x = \frac{1}{2} \log_3 x$

$\log_3 x + \frac{1}{2} \log_3 x = 12$

$\log_3 x^{\frac{3}{2}} = 12$

$3^{12} = x^{\frac{3}{2}}$

$x = 3^8$